A State Personal Income Tax Simulation Model

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INTRODUCTION

This paper describes a simple policy simulation model of the Alaska state personal income tax. The model provides estimates of the long-run revenue impact of changes in the tax schedule and, in conjunction with an econometric model of the state, estimates of long-run impact on economic growth.

Personal income tax simulation models fall into two general categories. The first is the micro-simulation model which contains a sample of actual returns allowing the researcher to recalculate the tax liability under different assumptions. The second is an aggregate model which typically divides returns into income categories, calculates effective rates for each category, and sums across categories to get total tax revenue. The former method allows for detailed examination of the revenue impact of tax law changes but is expensive and difficult to link to an econometric model. The latter is much less accurate for policy analysis, but is relatively inexpensive and easy to manipulate.

Unfortunately, neither approach is presently possible in the Alaska case because of the unavailability and unreliability of data on individual returns by income class. There is, however, a need for a model of the Alaska personal income tax because of its position as a major source of state revenue and because of the implications for state economic growth of changes in the personal income tax structure.

The model presented here is designed for long-run policy simulation in conjunction with an econometric model. Thus, the focus is on both the estimation of tax revenues and the general equilibrium effects of tax changes. Section I describes the model, and results of model testing are reviewed in Section II. In the final section, the model is incorporated into a long-run simulation model of the Alaskan economy and the policy simulation possibilities of the model are displayed.

I. THE MODEL

The essential feature of the model is that the liability for the average taxpayer is calculated based upon average taxable income and a single equation representation of the actual progressive tax schedule. The resulting average tax revenue is multiplied by the number of taxpayers to arrive at total tax revenue. This formulation of the

1Pechman [6].
2McLaren [5].
tax structure minimizes several of the problems arising from aggregate estimation of the relationship between aggregate income and tax liability. In particular, the rapid population growth Alaska has experienced since statehood complicates the relationship between aggregate taxable income and tax receipts.

This approach also allows, with a minimum number of equations, for a structural modeling of the tax to facilitate simulations of changes in the tax structure. In particular, changes in the base rates and progressivity of the tax schedule can be incorporated into the model.

The tax model equations are shown in Table 1. The Alaska personal income tax essentially "piggybacks" the federal tax structure in that deductions and exemption values are similar in both. Two exceptions to this general rule are that the federal employee cost of living allowance is state taxable income, and military wages and salaries is not. The state tax schedule is lower than the federal but progressive over a wide range of taxable incomes. For single taxpayers, the rate is 3 percent under $2,000 and rises to 14.5 percent of taxable income in excess of $150,000.

Alaska adjusted gross income is determined by the first three equations on the basis of the aforementioned adjustments to federal gross income. The number of returns, with a joint return counted as two, is determined by employment, while exemptions are a function of civilian population. Tax deductions are calculated on a per-return basis from adjusted gross income per return.

From taxable income per taxpayer, the calendar year tax liability is calculated using a double log specification. The coefficient on average taxable income of 1.42 calculated by regression analysis on annual data is interpreted as the elasticity of tax revenue to taxable income for the average individual taxpayer. The equation itself is a representation of the average tax rate schedule.

This equation can be compared to a similar relationship between taxable income and tax liability calculated directly from the published tax schedule which is as follows:

\[
\text{Log (tax liability)} = -3.83 + 1.34 \log \text{(Taxable income)}
\]

\[R^2 = .997\]

The higher elasticity in the equation estimated from historical data is consistent with the fact that the historical series is based upon the average relationship between taxable income and tax receipts aggregated across all taxpayers. Given that the average tax schedule is progressive, the average tax bill for any distribution of taxable incomes will be larger than the tax liability for the individual with taxable income equal to the average.

\[3\text{Singer [8].}\]
### Table 1
Alaska Income Tax Model $^a$

1. \[ \log(\text{Federal Adjusted Gross Income}) = 0.104 + 0.975 \times \log(\text{Personal Income}) \]
   \[ (0.60) \quad (52.04) \quad R^2 = 0.99 \quad (1961-1974) \]

2. Federal Cost of Living Adjustment = \( (1 - 1/(1 + \text{Cost of Living Adjustment Rate}) ) \times \)
   Federal Civilian Wages and Salaries

3. Adjusted Gross Income = Federal Adjusted Gross Income + Federal Cost of Living Adjustment -
   Military Wages and Salaries

4. \[ \log(\text{Tax Returns})^b = -0.813 + 0.907 \times \log(\text{Employment}) \]
   \[ (5.30) \quad (28.65) \quad R^2 = 0.99 \quad (1961-1972) \]

5. \[ \log(\text{Exemptions}) = -6.96 + 1.02 \times \log(\text{Civilian Population}) \]
   \[ (12.79) \quad (10.32) \quad R^2 = 0.90 \quad (1961-1974) \]

6. \[ \log(\text{Value of Deductions per Return}) = -1.923 + 1.2 \times \log(\text{Adjusted Gross Income per Return}) \]

7. Taxable Income = Adjusted Gross Income - Number of Personal Exemptions * (Value of Exemptions +
   EXEMPTION) - Value of Deductions

8. Taxable Income per Return = Taxable Income / Tax Returns

9. \[ \log(\text{Preliminary Tax Revenue per Return}) = -3.61 - \text{ABASERATE} + \]
   \[ (25.88) \quad (11.47) \quad R^2 = 0.93 \quad (1961-1972) \]
   \[ (1 - \text{ATAXRATE}) \times 1.42 \times \log(\text{Taxable Income per Return}) \]

10. Tax Revenue per Return = \( (1 - \text{TAXCREDIT}) \times \text{Preliminary Tax Revenue per Return} - \text{TAXCREDIT} \)

11. Calendar Year Tax Revenues = Tax Revenue per Return * Tax Returns

12. Fiscal Year Tax Revenues = \( 0.538 \times \text{Calendar Year Tax Revenues} \) \( \text{Lagged One Period} + 0.442 \times \)
    Calendar Year Tax Revenues

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**POLICY VARIABLES**

- ATAXRATE = Change in Tax Rate Progressivity
- ABASERATE = Change in Floor of Tax Rate Schedule
- EXEMPTION = Change in Value of Personal Exemption
- TAXCREDIT = Tax Credit
- TAXCREDIT = Tax Credit as Percent of Liability

**DATA SOURCES**

- State of Alaska, Department of Labor, unpublished data on employment and wages and salaries.
- State of Alaska, Department of Revenue, *Revenue Sources*, annual, and *Cumulative Summary of Revenue*, monthly.
Figure 1 illustrates the regressions run on the historical data and the average tax schedule. At a taxable income of $10,000, an individual would pay $470 in taxes according to the regression taken from the tax schedule. According to the historical relationship, however, an average taxable income of $10,000 would have resulted in an average return of $710. This is equivalent to the average return on a distribution of taxable incomes around an average of $10,000.4

If the size distribution of taxable incomes remains stable over time and other sources of error reflected by the difference between the intercept terms of the two regressions is unchanged, there will be a one-to-one correspondence between the schedule regression and the historical regression. This will allow one to perform approximate policy analysis of changes of the average tax rates and progressivity of the schedule directly.

For the purpose of long-run policy simulation in the Alaskan context, this approach seems advantageous because per capita income and population are the variables which will probably be most important in terms of the pattern of future growth, and it is thus important to structure the model around changes in those variables.

Finally, fiscal year tax liability is calculated from calendar year revenues. The former is of concern to government budget planners, while the latter is input into the larger simulation model. Calendar year revenues are one of the components of the difference between personal income and disposable personal income.

II. MODEL TESTING

The $R^2$ values and t statistics for coefficients were reported in Table 1. Evaluation of a set of equations cannot be done on the basis of a strict statistical test. Rather, the model can be "validated" through historical simulation and ex-post forecasting.5

An historical simulation of the model was done for the period 1967 through 1972, the last year for which a complete data series is available. The root mean square percent error (RMSPE) and mean absolute percent error (MAPE) for calendar year (RTISC) and fiscal year (RTIS) personal income tax collections are as follows:

<table>
<thead>
<tr>
<th></th>
<th>RMSPE</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RTIS</td>
<td>6.10</td>
<td>4.29</td>
</tr>
<tr>
<td>RTISC</td>
<td>3.96</td>
<td>3.32</td>
</tr>
</tbody>
</table>

4Some of the difference between regression lines is the result of different intercept terms. The historical regression intercept may incorporate the effect of relevant but unspecified variables.
5Pindyek and Rubinfeld [7–pp. 310-320].
Figure 1. Tax Liability as a Function of Taxable Income
The RMSPE weighs outlying observations more heavily than MAPE, while MAPE is the measure more commonly reported in regional models. Although there is no statistical test against which to evaluate the calculated RMSPE or MAPE, Glickman indicates that most econometricians are content if most variables in a model have a MAPE of 3 percent.\(^6\)

It is not possible to formally compare the MAPE statistics resulting from this simulation with other tax models because the only ones for which MAPE statistics are reported are imbedded within state econometric models.\(^7\) The tax components of these models have relatively large MAPEs, but the complete models are much more complex. Thus, there is more variability inherent in their simulations, and one could argue that accuracy in tracking revenue variables was sacrificed in order to reduce the tracking error of other aggregate variables. Considering this, the "noise" in the Alaska revenue series and the relatively small size of the population, the MAPE values do not seem unreasonable.

Another attempt to validate a model is to perform an ex-post forecast and this has been done for the years 1973-75 using actual and estimated values for the variables exogenous to the model. The results of this text are mixed.

<table>
<thead>
<tr>
<th>Calendar Year Revenues</th>
<th>Fiscal Year Revenues</th>
</tr>
</thead>
<tbody>
<tr>
<td>(RTISC)</td>
<td>(RTIS)</td>
</tr>
<tr>
<td></td>
<td>simulated</td>
</tr>
<tr>
<td>1973</td>
<td>54.657</td>
</tr>
<tr>
<td>1974</td>
<td>71.098</td>
</tr>
<tr>
<td>1975</td>
<td>113.733</td>
</tr>
<tr>
<td>Total</td>
<td>239.49</td>
</tr>
</tbody>
</table>

Compared to the period of smooth and gradual growth during the period of the historical simulation, the ex-post forecast covers a period of three years during which actual calendar year revenues nearly tripled. The simulated values for RTISC for each year are not close to the actual values, although the large increase in revenues is picked up and the total for the three-year period has only a 2

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\(^6\)Glickman [3--p. 165].

\(^7\)Friedlaender, Treyz and Tresch [2] and Delaware Econometric Modeling Group [1].
percent error. Since RTIS is a weighted average of calendar year totals, one would expect some error in this simulated series during periods of accelerated or decelerated growth. Over the cycle, however, that error would cancel out.

These validations of the model must be analyzed subjectively. A model that replicates the historic period well may simulate poorly because of improper or inadequate specification. This model replicates the historic period and is able to respond adequately to the period of rapid growth in the ex-post forecast period. It is not completely accurate for the forecast period but since the main purpose of the model is policy analysis, proper specification is relatively more important than minimal forecast error. The responsiveness of the model for the ex-post forecast period should thus be viewed as a successful test of the robustness of the model.

III. MODEL SIMULATIONS

The state of Alaska is in a position where it is capable of using fiscal policy to affect state economic growth. It can reduce the personal income tax, thereby increasing disposable personal incomes and effective demand for goods and services. This will increase employment opportunities and through resulting migration, population. In order to investigate the size of the potential impact of this fiscal policy, the personal income tax model was linked to an econometric model of the state through the exogenous variables in the tax model.\(^8\) The resulting model permits the analysis of both direct revenue effects and general economic effects of tax changes.

Two policy experiments were done. In the first, the value of a personal exemption is increased by $1,200 beginning in fiscal year 1978; and in the second, the tax schedule is modified. The base tax rate is reduced from 3 percent to 1 percent on the first $2,000 of taxable income, and the progressivity is reduced so that at $10,000, the marginal rate falls from 6 percent to approximately 3 percent and at $20,000, from 9 percent to approximately 5 percent. This is accomplished by a downward adjustment of the intercept term in the tax schedule equation. The new schedule of marginal tax rates can be calculated from the new shape of the schedule equation.\(^9\) In both experiments, the level of government expenditures is not reduced by the amount of the tax reduction. The lost revenues are recovered by drawing down the substantial general fund balance.

The initial direct tax revenue impacts are approximately equivalent for the two changes. By incorporation into a long run simulation model, the long-run revenue

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\(^8\)Kreage [4].

\(^9\)In equation 9, CRITSFCH takes a value of .5. The published tax schedule equation intercept is also reduced by .5 and a new marginal rate schedule calculated from it.
impact and effects on the pattern of economic growth can be traced through comparison to a base case.

Table 2 compares the simulation values, measured from the base case, for several variables. In either case, the increase in disposable personal income resulting from the tax cuts has stimulated demand and led to employment growth. This growth has generated tax revenue,\textsuperscript{10} partially offsetting the decline caused by the initial tax cuts. The long-run stimulative effect of the schedule reduction is stronger than of the exemption value increase because growth in incomes and the effect of inflation exceed population growth during the period of simulation.\textsuperscript{11} The net decline in general fund revenues is larger in the case of the schedule reduction because differentially greater population increase has caused larger demand for public expenditures. This, in turn, has caused the general fund balance to be drawn down more rapidly and has reduced interest earnings on that balance.

The change in the level of personal income tax revenues is net of two effects, the one being the reduction in the tax liability for an individual and the other being the increase in the number of taxpayers. These effects can be approximated by calculating the tax bill in each policy experiment using the original tax schedule. They are shown in Table 3. The difference between the original tax bill and the actual tax bill after the tax change is the amount attributable to the tax cut. The residual is attributable to economic growth. As expected, the economic growth component is larger for the schedule cut because of its larger stimulative effect. It is interesting to note, however, that in either case, the economic growth effect is fairly small compared to the size of the tax cut itself.

The simulation analysis indicates that as a result of Alaska’s dependence upon resource-based revenues, primarily petroleum, in combination with an open border, tax changes which affect disposable personal income have significant general equilibrium effects. For example, tax cuts have a negative impact on state revenues in spite of the fact that they generate economic growth, because the incremental revenues from growth are more than offset by general fund balance interest reductions.

\textsuperscript{10}The econometric model contains a complete state revenue and expenditure component. Other revenue sources sensitive to the level of endogenous economic activity are primarily the corporate income tax, business license tax, and motor fuels tax.

\textsuperscript{11}A steadily increasing exemption value change could result in an identical long-run effect measured in these aggregate terms, but might result in a more socially acceptable distribution of income.
Table 2
Simulation Results*

<table>
<thead>
<tr>
<th>Year</th>
<th>Fiscal Year Income Tax Revenues ($10^6)</th>
<th>Disposable Personal Income ($10^6)</th>
<th>Employment (10^3)</th>
<th>Total State General Fund Revenues ($10^6)</th>
<th>Change in General Fund Revenue Net of Income Tax Change ($10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>S</td>
<td>E</td>
<td>S</td>
<td>E</td>
</tr>
<tr>
<td>1979</td>
<td>-39.07</td>
<td>-47.40</td>
<td>145.69</td>
<td>184.74</td>
<td>5.01</td>
</tr>
<tr>
<td>1980</td>
<td>-42.69</td>
<td>-56.86</td>
<td>170.25</td>
<td>235.08</td>
<td>5.59</td>
</tr>
<tr>
<td>1982</td>
<td>-54.73</td>
<td>-88.35</td>
<td>240.38</td>
<td>396.07</td>
<td>7.12</td>
</tr>
<tr>
<td>1983</td>
<td>-60.14</td>
<td>-103.77</td>
<td>274.24</td>
<td>485.72</td>
<td>7.72</td>
</tr>
<tr>
<td>1984</td>
<td>-64.30</td>
<td>-115.18</td>
<td>290.71</td>
<td>521.15</td>
<td>7.91</td>
</tr>
</tbody>
</table>

* Measured as the difference from a base case

E = increase in exemption value
S = decrease in marginal tax schedule
<table>
<thead>
<tr>
<th>Year</th>
<th>Calendar Year Reduction in Personal Income Tax ($10^6)</th>
<th>Rate Change Component ($10^6)</th>
<th>Economic Growth Component ($10^6)</th>
<th>Reduction in Average Tax Bill ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>S</td>
<td>E</td>
<td>S</td>
</tr>
<tr>
<td>1978</td>
<td>-36.71</td>
<td>-43.71</td>
<td>-41.17</td>
<td>-47.69</td>
</tr>
<tr>
<td>1979</td>
<td>-40.77</td>
<td>-52.07</td>
<td>-45.50</td>
<td>-57.63</td>
</tr>
<tr>
<td>1980</td>
<td>-45.11</td>
<td>-62.90</td>
<td>-50.80</td>
<td>-70.10</td>
</tr>
<tr>
<td>1981</td>
<td>-52.55</td>
<td>-82.41</td>
<td>-60.15</td>
<td>-92.86</td>
</tr>
<tr>
<td>1982</td>
<td>-57.48</td>
<td>-95.86</td>
<td>-66.48</td>
<td>-109.16</td>
</tr>
<tr>
<td>1983</td>
<td>-63.50</td>
<td>-113.77</td>
<td>-74.19</td>
<td>-130.49</td>
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<tr>
<td>1984</td>
<td>-65.32</td>
<td>-116.96</td>
<td>-76.97</td>
<td>-135.34</td>
</tr>
<tr>
<td>1985</td>
<td>-67.90</td>
<td>-124.02</td>
<td>-80.27</td>
<td>-144.03</td>
</tr>
</tbody>
</table>

* Measured as the difference from a base case

E = increase in exemption value
S = decrease in marginal tax schedule
REFERENCES


